

Maximal acceleration and radiative processes

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We derive the radiation characteristics of an accelerated, charged particle in a model due to Caianiello in which the proper acceleration of a particle of mass m has the upper limit $\mathcal{A}_m = 2mc^3/\hbar$. We find two power laws, one applicable to lower accelerations, the other more suitable for accelerations closer to \mathcal{A}_m and to the related physical singularity in the Ricci scalar. Geometrical constraints and power spectra are also discussed. By comparing the power laws due to the maximal acceleration with that for particles in gravitational fields, we find that the model of Caianiello allows, in principle, the use of charged particles as tools to distinguish inertial from gravitational fields locally.

INTRODUCTION

In a model aimed at providing quantum mechanics with a geometrical framework [1–3], Caianiello introduced the upper limit $\mathcal{A}_m = 2mc^3/\hbar$ to the proper acceleration of a particle. This mass dependent limit, or maximal acceleration (MA), can be derived from quantum mechanical considerations and the fact that the acceleration is largest in the particle rest frame [4–6]. The absolute value of the proper acceleration therefore satisfies the inequality $a \leq \mathcal{A}_m$. No counterexamples are known to the validity of this inequality.

Classical and quantum arguments supporting the existence of a MA have been given in the literature [7–36]. MA also appears in the context of Weyl space [37–40] and of a geometrical analogue of Vigier’s stochastic theory [41]. It is invoked as a tool to rid black hole entropy of ultraviolet divergences [42] and is at times regarded as a regularization procedure [43] that avoids the introduction of a fundamental length [44], thus preserving the continuity of space-time.

An upper limit on the acceleration also exists in string theory [45–49] when the acceleration induced by the background gravitational field reaches the critical value $a_c = \lambda^{-1} = (m\alpha)^{-1}$ where λ , m and α^{-1} are string size, mass and tension. At accelerations larger than a_c the string extremities become casually disconnected. Frolov and Sanchez [50] have found that a universal critical acceleration must be a general property of strings. It is the same cut-off required by Sanchez in order to regularize the entropy and the free energy of quantum strings [51].

Applications of Caianiello’s model include cosmology [52–54], the dynamics of accelerated strings [55, 56] and neutrino oscillations [57–59] and the determination of a lower neutrino mass bound [60]. The model also makes the metric observer-dependent, as conjectured by Gibbons and Hawking [61].

The model has been applied to classical [62] and quantum particles [63] falling in the gravitational field of a collapsing, spherically symmetric object described by the Schwarzschild metric and also to the Reissner-Nordström [64] and Kerr [65] metrics. In the model, the end product of stellar collapse is represented by compact, impenetrable astrophysical objects whose radiation characteristics are similar to those of known bursters [66].

The consequences of the model for the classical electrodynamics of a particle [67], the mass of the Higgs boson [68, 69] and the Lamb shift in hydrogenic atoms [70] have been worked out. MA effects in muonic atoms [71] and on helicity and chirality of particles [72] have also been investigated.

Most recently Rovelli and Vidotto in two important works have found evidence for MA and singularity resolution in covariant loop quantum gravity [73],[74].

Caianiello’s model is based on an embedding procedure [62] that stipulates that the line element experienced by an accelerating particle is represented by

$$d\tau^2 = \left(1 + \frac{g_{\mu\nu}\ddot{x}^\mu\ddot{x}^\nu}{\mathcal{A}_m^2}\right) g_{\alpha\beta}dx^\alpha dx^\beta = \left(1 + \frac{a^2(x)}{\mathcal{A}_m^2}\right) ds^2 \equiv \sigma^2(x)ds^2, \quad (1)$$

where $g_{\alpha\beta}$ is a background gravitational field. The effective space-time geometry given by (1) therefore exhibits mass-dependent corrections that in general induce curvature and violations of the equivalence principle.

The purpose of this paper is to determine the radiation characteristics of an accelerating charged fermion in the field of the metric (1) in the process described by Fig.1. We tackle the problem in two distinct ways, first by using solutions of the covariant Dirac equation that are exact to first order in the metric deviation $\gamma_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the metric of flat space-time (of signature -2). In these relativistic solutions the MA field of (1) appears in a phase factor and the Riemann tensor is linearized. This approach is suitable to study the effects of MA when $a \ll \mathcal{A}_m$. In the second approach, more suitable for extreme accelerations, we consider the effects of the Ricci scalar and of its

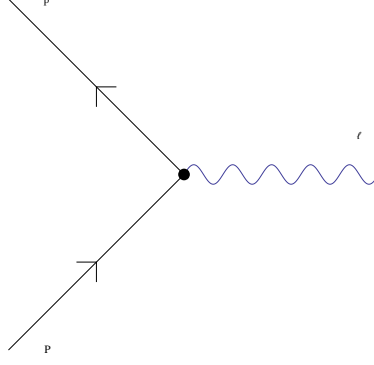


FIG. 1: This process, normally forbidden by conservation of energy and momentum, is allowed in the MA model.

physical singularity at $a = \mathcal{A}_m$ on the index of refraction of the incoming fermion. In this way some aspects of the problem at more elevated accelerations can be considered. The single-vertex diagram selected represents, potentially, the largest contribution to the radiation process. It is however forbidden kinematically in Minkowski space. Let us consider in fact the process

$$P_\mu = p'_\mu + \ell_\mu, \quad (2)$$

in which an incoming massive particle of momentum P_μ and dispersion relation $P_\mu P^\mu = m_1^2$ produces a photon of momentum ℓ_μ $\ell^\mu = 0$, while the outgoing particle has momentum p'_μ $p'^\mu = m^2$. Conservation of energy-momentum requires $P_\mu = p'_\mu + \ell_\mu$. In the rest frame of P we have $0 = \vec{p}' + \vec{\ell}$, which gives $\vec{\ell} = -\vec{p}'$, $P_0 = m_1$ and again $m_1 = \sqrt{\ell^2 + m^2} + \ell$. Then $(m_1 - \ell)^2 = \ell^2 + m^2$ leads to $\ell = (m_1^2 - m^2)/2m_1$ which shows that for $m_1 = m$, the case considered, we get $\ell = 0$ and the process becomes physically meaningless. There are processes, however, in which massive particles emitting a photon are not kinematically forbidden. This is certainly the case when gravitation alters the dispersion relations of at least one of the particles involved [75, 76]. This is also the case with MA [77].

THE DIRAC EQUATION AND MA

Because MA acts as a gravitational field according to (1), we can also expect changes in the index of refraction and dispersion relations of the incoming, accelerating fermion.

Let us consider, for simplicity, the case of hyperbolic motion in which the fermion always experiences a constant acceleration $a_\mu a^\mu = 1/\xi^2$. From (1) we immediately obtain the modified Minkowski metric

$$d\tau^2 = \left(1 - \frac{1}{\mathcal{A}_m^2 \xi^2}\right) (dt^2 - dx^2 - dy^2 - dz^2), \quad (3)$$

for that region of space-time delimited by the physical branch of the hyperbola $t^2 - z^2 \equiv \xi^2$. The behavior of spin-1/2 particles in the presence of a gravitational field $g_{\mu\nu}$ is determined by the covariant Dirac equation

$$[i\gamma^\mu(x)\mathcal{D}_\mu - m]\Psi(x) = 0, \quad (4)$$

where $\mathcal{D}_\mu = \nabla_\mu + i\Gamma_\mu(x)$, ∇_μ is the covariant derivative, $\Gamma_\mu(x)$ the spin connection and the matrices $\gamma^\mu(x)$ satisfy the anti-commutation relations $\{\gamma^\mu(x), \gamma^\nu(x)\} = 2g^{\mu\nu}(x)$. Both $\Gamma_\mu(x)$ and $\gamma^\mu(x)$ can be related to the usual constant Dirac matrices $\gamma^{\hat{\alpha}}$ by using the tetrad fields $e_{\hat{\alpha}}^\mu$ and the relations

$$\gamma^\mu(x) = e_{\hat{\alpha}}^\mu(x)\gamma^{\hat{\alpha}}, \quad \Gamma_\mu(x) = -\frac{1}{4}\sigma^{\hat{\alpha}\hat{\beta}}e_{\hat{\alpha}}^\nu e_{\nu\hat{\beta};\mu}, \quad (5)$$

where $\sigma^{\hat{\alpha}\hat{\beta}} = \frac{i}{2}[\gamma^{\hat{\alpha}}, \gamma^{\hat{\beta}}]$. A semicolon and a comma are frequently used as alternative ways to indicate covariant and partial derivatives respectively. Then the connection is represented by

$$\Gamma_\mu = \frac{1}{2}\sigma^{\hat{\alpha}\hat{\beta}}\omega_{\mu\hat{\alpha}\hat{\beta}}, \quad (6)$$

where

$$\sigma^{\hat{\alpha}\hat{\beta}} = \frac{1}{4} [\gamma^{\hat{\alpha}}, \gamma^{\hat{\beta}}], \quad \omega_{\mu}^{\hat{\alpha}\hat{\beta}} = (\Gamma_{\mu\nu}^{\lambda} e_{\lambda}^{\hat{\alpha}} - \partial_{\mu} e_{\nu}^{\hat{\alpha}}) e_{\hat{\beta}}^{\nu} \quad (7)$$

$$\Gamma_{\mu\nu}^{\lambda} = \frac{1}{2} g^{\lambda\alpha} (g_{\alpha\mu,\nu} + g_{\alpha\nu,\mu} - g_{\mu\nu,\alpha}), \quad \gamma^{\mu}(x) = e^{\mu}_{\hat{\alpha}} \gamma^{\hat{\alpha}}, \quad e^{\mu}_{\hat{\alpha}} e_{\mu}^{\hat{\beta}} = \delta_{\hat{\alpha}}^{\hat{\beta}}. \quad (8)$$

The form of (1) determines the tetrad field and the connection. From (3)-(8) we find

$$e_{\mu}^{\hat{\alpha}}(x) = \sigma(x) \delta_{\mu}^{\hat{\alpha}}, \quad \Gamma_{\mu} = \sigma^{\hat{\alpha}\hat{\beta}} \eta_{\hat{\alpha}\mu}(\ln \sigma)_{,\hat{\beta}}. \quad (9)$$

The solution of (4) exact to first order in $\gamma_{\mu\nu} = f\eta_{\mu\nu}$ where $f = \sigma^2 - 1$, is [78–80]

$$\Psi(x) = -\frac{1}{2m} (-i\gamma^{\mu}(x)\mathcal{D}_{\mu} - m) e^{-i\Phi_T} \Psi_0(x), \quad (10)$$

where $\Phi_T = \Phi_s + \Phi_G$ is of first order in $\gamma_{\alpha\beta}(x)$. The factor $-1/2m$ on the r.h.s. of (10) is required by the condition that both sides of the equation agree when the MA contribution vanishes. Similar solutions can also be found for all covariant wave equations [81–84]. We choose $\Psi_0(x) \propto e^{-ip_{\alpha}x^{\alpha}}$ and drop carets in what follows. In (10) $\Phi_S(x) = \mathcal{P} \int_P^x dz^{\lambda} \Gamma_{\lambda}(z)$ and

$$\Phi_G(x) = -\frac{1}{4} \int_P^x dz^{\lambda} [\gamma_{\alpha\lambda,\beta}(z) - \gamma_{\beta\lambda,\alpha}(z)] ((x^{\alpha} - z^{\alpha})p^{\beta} - (x^{\beta} - z^{\beta})p^{\alpha}) + \frac{1}{2} \int_P^x dz^{\lambda} \gamma_{\alpha\lambda} p^{\alpha}. \quad (11)$$

It follows from (4) and (11) that the physical momentum of the incoming fermion is

$$P_{\mu} = -p_{\mu} - \Phi_{G,\mu} = -p_{\mu} - \frac{1}{2} \gamma_{\alpha\mu}(x) p^{\alpha} + \frac{1}{2} \int_P^x dz^{\lambda} (\gamma_{\mu\lambda,\beta}(z) - \gamma_{\beta\lambda,\mu}(z)) p^{\beta}. \quad (12)$$

For hyperbolic motion in the (t, z) -plane we find $P_3 = -p_3 - \Phi_{G,3} = -p_3 - \frac{p_0^2 f}{2p}$, where $p \equiv p^3$. It therefore follows that the particle's momentum remains finite even at the MA limit $f = -1$, and so does the index of refraction

$$N_f = \frac{|\vec{P}|}{P_0} = \frac{y}{\sqrt{1+y^2}} \left\{ \frac{y^2 - \frac{f}{2}(1+y^2)}{y^2 + \frac{f}{2}(1+y^2)} \right\}, \quad (13)$$

where $y \equiv p/m$. It also follows that $N_f \sim 1$ for $f = 0$ and that there are no allowable values of f for which N_f vanishes. In addition N_f diverges for values of y and f such that $y^2 = -f/2/(1+f/2)$. All these results also apply to self-accelerating particles. Though obtained by linearizing (4), solution (10) can be extended to any order in $\gamma_{\mu\nu} = f\eta_{\mu\nu}$.

THE POWER EMITTED

It now is possible to calculate the power radiated as photons in the process of Fig.1 following the procedure outlined in [75, 76]. We find

$$W = \frac{1}{8(2\pi)^2} \int \delta^4(P - p' - l) \frac{|M|^2}{P^3} \Theta(p'_0) \delta(p'^2 - m^2) d^4 p' d^3 \ell, \quad (14)$$

where

$$|M|^2 = Z^2 e^2 \left[-4(p'_{\alpha} P^{\alpha}) + 8(p_{\alpha} P^{\alpha}) \right] + 2 \left[2\gamma_{\alpha\beta} p'^{\alpha} p^{\beta} + \gamma_{\alpha\beta} \eta^{\alpha\beta} (-p'_{\lambda} p^{\lambda}) + m^2 \right], \quad (15)$$

and, neglecting the spin-MA interaction, $P_{\alpha} = p_{\alpha} + \Phi_{G,\alpha}$. Using the identity $\int \frac{d^3 p'}{2p'_0} = \int d^4 p' \Theta(p'_0) \delta(p'^2 - m^2)$, integrating over $d^4 p'$ and writing the on-shell condition for p' as

$$\delta(\cos \theta - \frac{m^2 - P_{\alpha} P^{\alpha} + 2\ell_0 P_0}{2|\vec{P}||\vec{\ell}|}), \quad (16)$$

we obtain

$$W = \frac{(Ze)^2}{8\pi} \int d\ell \frac{\ell}{P|\vec{P}|} \left| \frac{f-1}{2} (P_\alpha P^\alpha) - \frac{m^2}{2} (f+1) + (f+2)(p_\alpha P^\alpha) \right|, \quad (17)$$

where $P^3 \equiv P$. Using (12) to calculate the components of P_μ and substituting the results in (17) we obtain

$$W \approx \frac{5(Ze)^2}{32\pi} \frac{m^2 \ell^2}{p^2} \left| f \left(1 + \frac{f p_0^2}{p^2} \right) \right| = \frac{5(Ze)^2}{32\pi} \frac{\ell^2}{y^2} \left| f + \left(1 + \frac{1}{y^2} \right) f^2 \right|. \quad (18)$$

Note that (18) vanishes as $f \rightarrow 0$ (vanishing acceleration) and diverges as $y \rightarrow 0$. This infrared divergence has already been discussed in [75] and can be removed [85]. The power spectrum follows immediately from (18) and is

$$\frac{dW}{d\ell} = \frac{5(Ze)^2}{16\pi} \frac{\ell}{y^2} \left| f + \left(1 + \frac{1}{y^2} \right) f^2 \right|. \quad (19)$$

These results require that the inequalities $-1 \leq \cos \theta \leq 1$ be satisfied. We find that they can be both satisfied if

$$\frac{m^2 f}{2p_0 + 2p + \frac{m^2 f(m^2 + 2p^2)}{p^2 p_0}} \leq \ell \leq \frac{m^2 f}{2p_0 - 2p + \frac{m^2 f(m^2 + 2p^2)}{p^2 p_0}}. \quad (20)$$

In the high momentum approximation $p \gg m$, (20) becomes $0 \leq \ell \leq pf/(1+3f/2)$. The condition that $\ell > 0$ requires in addition that $f < -2/3$. By substituting $f \sim -1$ and $\ell \sim 2p$ in the expression for $\cos \theta$ we find $\cos \theta \sim 1$ and radiation is in the forward direction. At the other extreme, $\ell \sim 0$, we also find $\theta \sim 0$. In the approximation used, radiation is therefore possible in the interval of accelerations $-1 < f < -2/3$.

THE EFFECT OF THE CURVATURE SCALAR

A better perspective on the behaviour of a fermion closer to the limit \mathcal{A}_m can be acquired by introducing a different approximation to the dispersion relation of the fermion in the MA field. Let us therefore multiply (4) on the left with $(-i\gamma^\nu(x)D_\nu - mc/\hbar)$. We obtain the second-order equation

$$\left(\gamma^\mu(x)\gamma^\nu(x)D_\mu D_\nu + \frac{m^2 c^2}{\hbar^2} \right) \psi(x) = 0, \quad (21)$$

which, on using the relations $[D_\mu, D_\nu] = -\frac{i}{4}\sigma^{\alpha\beta}R_{\alpha\beta\mu\nu}$ and $\sigma^{\mu\nu}\sigma^{\alpha\beta}R_{\mu\nu\alpha\beta} = 2R$, reduces to

$$\left(g^{\mu\nu}D_\mu D_\nu - \frac{R}{4} + \frac{m^2 c^2}{\hbar^2} \right) \psi(x) = 0. \quad (22)$$

In the eikonal approximation $\nabla_\mu \psi = ip_\mu \psi$, (22) yields

$$-g^{\mu\nu}p_\mu p_\nu - \frac{R}{4} + m^2 = 0. \quad (23)$$

From this relation one obtains the index of refraction

$$n_f = \frac{|\mathbf{p}|}{p_0} = \sqrt{g^{00}} \sqrt{1 - \frac{m^2 - R/4}{g^{00}p_0^2}}, \quad (24)$$

which contains the scalar curvature R and its physical singularity at $f = -1$ and therefore differs substantially from (13). In (24) $|\mathbf{p}| = \sqrt{|g^{ij}p_i p_j|}$ and the radiation angle between \mathbf{p} and ℓ momenta is given by,

$$\cos \theta' = \frac{g^{ij}p_i \ell_j}{\sqrt{g^{ij}p_i p_j} \sqrt{\eta^{ij}\ell_i \ell_j}}. \quad (25)$$

Multiplying (2) by p_μ and ℓ_μ , and using the dispersion relations (23) and $\ell_\mu \ell^\mu = 0$, we get

$$m^2 - \frac{R}{4} = g^{\mu\nu}p_\mu p'_\nu + g^{\mu\nu}p_\mu \ell_\nu, \quad g^{\mu\nu}p_\mu \ell_\nu = g^{\mu\nu}p'_\mu \ell_\nu. \quad (26)$$

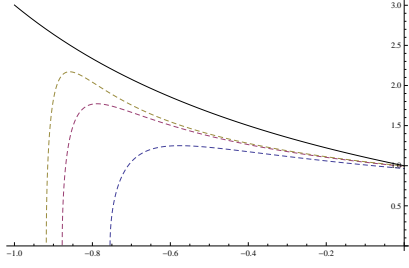


FIG. 2: The dotted curves refer to n_f and, respectively, to $y = 30, 20, 10$ from the top down. The upper curve refers to N_f for $k = 30$ and is not particularly sensitive to the values of y in the range considered.

On the other hand, assuming that after the emission of the photon, the fermion propagates with vanishing four-acceleration, the multiplication of (2) by p'_μ and the dispersion relation $\eta^{\mu\nu} p'_\mu p'_\nu = m^2$ give

$$g^{\mu\nu} p'_\mu p'_\nu = m^2 + g^{\mu\nu} p'_\mu \ell_\nu. \quad (27)$$

Equations (26) and (27) lead to the result

$$g^{\mu\nu} p'_\mu \ell_\nu = -\frac{R}{8}, \quad R = \frac{6\mathcal{A}_m^2 f^3}{(1+f)^3}. \quad (28)$$

The angle (25) therefore becomes

$$\cos \theta' = \frac{-g^{00} - \frac{R}{8\ell_0 p_0}}{n_f \sqrt{g^{00}}}. \quad (29)$$

It is convenient, in view of the following discussion, to write the explicit expression for the index of refraction (24)

$$n_f = \frac{1}{\sqrt{1+f}} \sqrt{1 - \frac{1+f}{y^2} \left(1 + \frac{6}{(1+f)^3}\right)}, \quad (30)$$

which gives $n_f = \sqrt{1 - (1 + 3/2)/y^2} \sim 1$ for $f = 0$ and high values of y . We also find that n_f is always positive for

$$f \geq \frac{1}{3}(-3 + y^2) + \frac{y^4}{3(-81 + y^6 + 9\sqrt{81 - 2y^6})^{\frac{1}{3}}} + \frac{1}{(-81 + y^6 + 9\sqrt{81 - 2y^6})^{\frac{1}{3}}}. \quad (31)$$

There also are two complex conjugate solutions of $n_f = 0$ that suggest complex optical properties for MA space-time. We will not investigate them in this work.

A comparison of the general behaviour of n_f and N_f is given in Fig. 2.

The calculation of the diagram of Fig.1 now proceeds as for (14) and (15), where P_3 is now replaced by p_3 , and use is made of (23)-(30). We obtain

$$W' = \frac{(Ze)^2 \ell^2}{32\pi y^2} \left| \frac{y\sqrt{1+f}}{\sqrt{(1+f)(1+y^2) + \frac{6f^3}{(1+f)^3}} - 1} - 1 \right|, \quad (32)$$

which vanishes for $f = 0$ and has singularities for f and y satisfying the equation $(1+f)(1+y^2) + 6f^3/(1+f)^3 = 1$. The infrared divergence for $y \rightarrow 0$ can also be removed [85]. The power spectrum $\frac{dW'}{d\ell}$ follows from (32).

Using (25), condition $-1 \leq \cos \theta' \leq 1$ becomes

$$\ell_1 \equiv \frac{R}{8\sigma p_0(n_f - \sigma)} \leq \ell \leq \frac{-R}{8\sigma p_0(n_f + \sigma)} \equiv \ell_2. \quad (33)$$

The l.h.s. of (33) is negative because $R < 0$ and $n_f \geq \sigma$, therefore ℓ_1 must be replaced by 0. When ℓ_2 is substituted in (25), we find $\cos \theta' \sim -R/(8m\sigma^2 y^-)$ where $y^- = \sqrt{(1 - 6f^3/(1+f)^3)/(1+f)}$ is the minimum value of y for which n_f is real. $\cos \theta'$ has the maximum value 0.22 at $f \sim 0.6$, it therefore follows that $\theta' \leq 0.43\pi$. W and W' are compared in Fig.3.

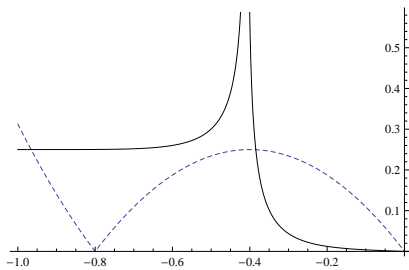


FIG. 3: The dotted and continuous curves refer to $W/\frac{Z^2 e^2 \ell^2}{32\pi}$ and $W'/\frac{Z^2 e^2 \ell^2}{32\pi}$, respectively. In both instances $y = 2$. The singularity in W' is shifted to the left toward a smaller f for higher values of y .

CONCLUSIONS

We have derived the radiation characteristics of a charged particle subject to acceleration regimes described by (1). A process described by the lowest order diagram of Fig.1 is kinematically forbidden unless the dispersion relations of anyone of the particles involved is altered either by a medium, or by a gravitational field. The incoming particle "feels", in the case considered, a gravitational field given by (1). We have therefore calculated the index of refraction due to MA which implies the metric structure (1) and can therefore be treated as a gravitational field. The calculation follows two distinct approaches. The first one, more suitable for low-acceleration regimes, is based on the method discussed in [78–80] that leads to solutions of the covariant Dirac equation that are exact to $\mathcal{O}(\gamma_{\mu\nu})$. In this approach the momentum remains finite, even in the case of a self-accelerating particle, but N_f has singularities when $f = -2y^2/(1+y^2)$. The spectrum is typically $\propto \ell$ and the infrared divergence of W for $y \rightarrow 0$ is of no consequence. In addition, if $p \gg m$, the process is possible if $f < -2/3$ and radiation is in the forward direction.

In the second approach we take into account the physical singularity in the Ricci scalar R by using the second order differential equation that can be obtained from the Dirac equation. This approach is better suited to study the high-acceleration regime. The corresponding index of refraction n_f has singularities when $(1+f)(1+y^2)+6f^3/(1+f)^3 = 1$. Radiation is possible in this case if $0 < \ell \leq \ell_2$ and is contained in the cone of aperture $\theta' \leq 0.43\pi$.

In the effective geometry (1), charged, accelerated fermions radiate according to (14) and (32). On the other hand fermions in "true" gravitational fields also emit radiation in the process represented by Fig.1[75]. It is in fact the index of refraction that by altering the dispersion relations of the fermion, and of any other particle in general, makes the process possible in vacuo. Note that unlike the case of Čerenkov radiation, it is not required that the photon be emitted in a medium.

The radiation power laws differ according to whether one considers the low-acceleration, or the high-acceleration regimes. By comparing W and W' with the power radiated by a fermion in a gravitational field [75]

$$W_{GF} = (Ze)^2 \frac{GM}{b} \frac{\ell^2}{y^2}, \quad (34)$$

where b is the impact parameter of the source, we find that in both instances GM/b is replaced by factors of f , while the dependence on $(\ell/y)^2$ remains the same. Quantum mechanics is, of course, formulated in terms of particle momenta and one therefore expects a dependence on y in (34), as well as in (14) and (32). Note, however that for $b \sim R$, the factor GM/R is a characteristic of the source, while f itself depends on m via \mathcal{A}_m and changes essentially from particle to particle. For $GM/b \sim f$ the radiation emitted by the charged fermion is such that $W_{GF} \simeq W$, while $W' \simeq W_{GF}$ if $GM/b \sim |-1 + y\sqrt{1+f}/\sqrt{(1+f)(1+y^2)+6f^3/(1+f)^3-1}| \sim 1$ at extreme accelerations, provided f applies in each case to the same particle. It therefore follows, from a comparison of W and W' with (34), that *Caianiello's model allows, in principle, the use of charged particles as tools to distinguish inertial from gravitational fields locally*. This is an interesting feature of the model that is not permissible in a purely classical theory [86, 87]. The result is a direct consequence of the quantum violations of the equivalence principle that MA introduces. These violations disappear when $\hbar \rightarrow 0$.

Because the role of the gravitational field is so essential in the process of Fig.1, it is expected that the procedures outlined in this work and in [75, 76] will be of use in extended theories of gravity [88].

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